Topology Optimization of a Magnetic Resonator Using Finite Difference Time Domain Method for Wireless Energy Transfer

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In this paper, a magnetic resonator is optimized based on electromagnetic wave analysis. To analyze the magnetic resonator, finite difference time domain (FDTD) method is used with Gaussian pulse source and perfectly matched layers. Topology optimization of a magnetic resonator is conducted for maximizing magnetic energy. After those approaches, the fast Fourier transform (FFT) is used to obtain the response of the magnetic resonator system over wide range of frequencies.

***Index Terms*—Topology optimization, finite difference time domain, magnetic resonator.**

I. INTRODUCTION

RECENTLY, electric vehicle have been the most important research topic because fossil fuel have been depleted. In order to retain the durable power sources, many researchers have studied and designed magnetic resonators for wireless energy transfer based on experiments, or the electronic circuit analysis [1], however, its efficiency is still low. Therefore, the design technique based on electromagnetic wave field analysis is also necessary to design a magnetic resonator for maximizing wireless energy transfer efficiency which is the ratio of the energy from transmitter to receiver.

In this research, 2D FDTD method which is the one of wave field analysis methods is used to analyze resonator, and topology optimization of a magnetic resonator is carried out to maximize magnetic energy for wireless energy transfer. After that, FFT is used to obtain the response of the magnetic resonator system over a wide range of frequencies.

II. FINITE DIFFERENCE TIME DOMAIN METHODS IN 2D

A. *Governing Equations*

The derivation of the FDTD update equations in electromagnetic wave problem starts from Faraday's law in (1) and Ampere's law in (2) of Maxwell's equations [2]:

$$-\mu\frac{\partial \mathbf{H}}{\partial t} = \nabla \times \mathbf{E} \tag{1}$$

$$\sigma\mathbf{E} + \varepsilon\frac{\partial \mathbf{E}}{\partial t} = \nabla \times \mathbf{H} \tag{2}$$

where $\mathbf{E}$, $\mathbf{H}$, μ, ε, σ, t are the electric field intensity, the magnetic field intensity, the permeability of material, the permittivity of material, the conductivity of material and time, respectively.

B. *FDTD Implementation in 2D Transverse Magnetic Mode*

In this work, the 2D transverse magnetic (TM) mode is used. Faraday's and Ampere's laws change as follow.

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\right) \tag{3}$$

$$\sigma E_x + \varepsilon\frac{\partial E_x}{\partial t} = \left(\frac{\partial H_z}{\partial y}\right) \tag{4}$$

$$\sigma E_y + \varepsilon\frac{\partial E_y}{\partial t} = \left(-\frac{\partial H_z}{\partial x}\right) \tag{5}$$

By replacing the derivatives in (3), (4), and (5) with central finite differences, 2D TM mode FDTD update equations are obtained.

A Gaussian pulse is used as a source in this work and it is a function of the following form as (6).

$$f(t) = ae^{-\frac{(t-b)^2}{2c^2}} \tag{6}$$

where a, b, c are arbitrary real constants and $e \approx 2.71828...$ (Euler's number). In this work, the parameter a is the height of the curve's peak, b is the position of the center of the peak and c controls the width of the peak. In this work, $f(t) = e^{-\frac{(t-60)^2}{400}}$ is used as a Gaussian pulse source, and it can consider the responses of 0~200MHz frequency range.

The size of area that can be simulated using FDTD is limited by computer memory and if nothing were done to outside of analysis domain, unintentional wave is reflected. To resolve this problem, a perfectly matched layer (PML) is the one of absorbing boundary conditions [3].

III. TOPOLOGY OPTIMIZATION IN EM WAVE PROBLEM

The study of the topology optimization on the electromagnetic system began several years ago. The principle of the topology optimization on electromagnetic systems is same with that of structural systems [4].

In electromagnetic wave problem, permittivity, permeability, and conductivity are the properties that can decide what material is. In this work, the objective funtion should be selected as a function of the design variables, and widely used interpolation scheme is called the Solid Isotropic Material with Penalization (SIMP). The SIMP type material model is defined as following equations,

$$\sigma = \sigma_0\rho^p \tag{7}$$

where σ_0 is the reference conductivity, respectively. ρ is a pseudo-density describing the amount of material in each point of the domain which can assume values between 0 and 1, and p is a penalization factor.

IV. NUMERICAL EXAMPLE

The proposed method is applied to a numerical example which is a simplified magnetic resonator. Fig. 1 shows initial design of a magnetic resonator which can be regarded as an energy harvesting device. The objective is to maximize the magnetic power sum from the objective value of first time iteration to the objective value of the last time iteration T=600:

$$\text{Minimize} \quad \text{Magnetic Energy Density}=\int_0^T \int_\Omega \frac{1}{2}\mu H^2 d\Omega dt \tag{8}$$

$$\text{Subject to} \quad \frac{\sum_1^N \rho_i V_i}{V_{initial} V_{fraction}} - 1 \le 0 \quad (0 \le \rho_i \le 1,\ i = 1,2,\ldots,n) \tag{9}$$

where Ω is the objective domain, f is objective function as magnetic energy, N is the number of design variables which are 675, ρ_i is the material density in i-th element, and V_i is the volume of the material in i-th element, $V_{initial}$ and $V_{fraction}$ are total initial volume and volume fraction of the design domain, respectively.

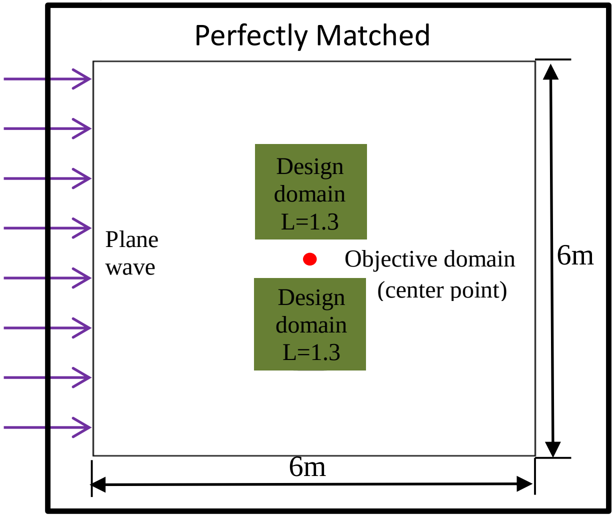
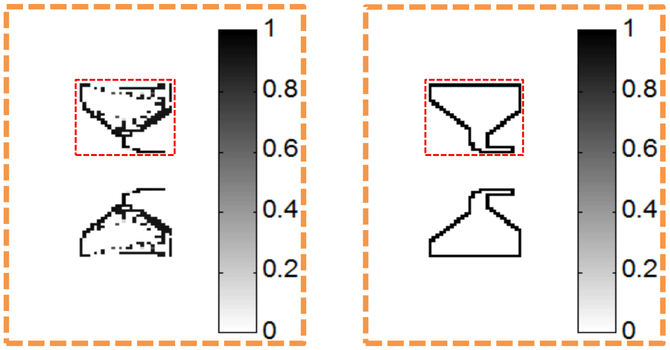


Fig. 1. Initial design of a magnetic resonator

Spatial step size is 0.3m and temporal step size is 8.34e-11s. Those are decided by aforementioned stability/courant conditions, and following Gaussian pulse is used as a plane wave source which propagates from left to right. The topology optimization starts from the initial design variables, objective value, and design sensitivities of design variables. The updated design variables are obtained from method of moving asymptotes (MMA) algorithm which is generally used in topology optimization problem [5].

The objective value of the initial design is 76.1839(when all design variables equal to 1). After 77 iterations, the optimized design obtained in Fig. 2. (a), and the objective value is 101.4324[J/m^3s]. It has increased by 33% from the initial design result. The two square pillar shapes of initial design domains were proper to obtain a conceptual design of the magnetic resonator through topology optimization. Note that topology optimization is useful to obtain an initial conceptual design.

It is difficult to manufacture the result shape of the topology optimization. So, it needs to redesign as similar as possible with optimized result. The redesign model of a magnetic resonator is shown in Fig. 2. (b). In that case, the results is 95.0002[J/m^3s], and it has increased by 24% from the initial design result. In order to obtain the frequency response function (FRF) from the results, FFT is used and the FRF results are shown in Fig. 3. We can obtain the objective values in 0~200MHz frequencies.



(a) (b)

Fig. 2. Optimized magnetic resonators for maximizing magnetic energy density: (a) Topology optimization result, (b) The optimized model for manufacture.

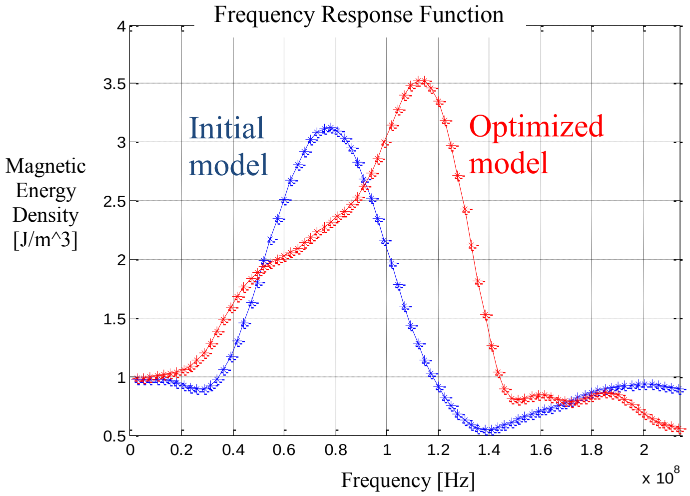


Fig. 3. Magnetic energy density: frequency response functions of initial model and optimized model

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